Applying Strategies From the Directed Reading Activity to a Directed Mathematics Activity

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A recent report produced by the Educational Testing Service (Dossey, Mullis, Lindquist, & Chambers, 1988) interpreting the results of the 1986 National Assessment of Educational Progress suggested that American children showed signs of improvement in mathematical proficiency at all three levels measured (ages 9, 13, and 17). This improvement, however, was primarily in the areas of lower level skills and basic concepts; children continue to demonstrate difficulty in applying mathematics to solve everyday problems as evidenced by their poor performance on word problems on the nationwide assessment test.

Though educators have long been aware of children's difficulties in solving word problems, what has been advocated as basic processes for problem solving have not changed dramatically. For example, Polya (1945) outlined the following: (a) understanding the problem, (b) devising a plan, (c) carrying out the plan, and (d) looking back. A widely adopted mathematics textbook for grade five (Eicholz, O'Daffer, & Fleenor, 1978) provided this model: (a) understand the question, (b) find the needed data, (c) plan what to do, (d) find the answer, and (e) check back. Uprichard, Phillips, and Soriano (1984) specified the following dimensions as an effective problem-solving strategy: (a) reading to understand the problem statement, (b) analyzing to organize and transform the input data in order to develop a plan, (c) estimating to test the adequacy of the plan, (d) translating to a mathematical language, (e) computing to find a possible numerical solution, and (f) verifying to compare the estimate to the solution and checking computation.

For the last generation, word problem-solving processes have remained relatively constant. Often when they are presented to the students, little or no follow up is planned to see that the students are actually using the process or even if they have the understanding to follow the process. For example, little or no mention has been made of the importance of understanding the vocabulary or applying reading comprehension skills except in an implied global sense--understand the problem/question. Specific strategies need to be taught and teacher directed in order for students to understand the problem--the same strategies that are used to develop comprehension of reading story selections.

In analyzing the characteristics of reading and mathematics, many similarities exist. Both are abstract, symbolic processes that are systems exclusive to humans. In addition, they are both complex processes that require a working knowledge of the interaction of numerous discrete skills.

Why do many students who score on or above grade level on reading tests and tests of computational skills perform below grade level on tests which measure mathematical problem solving? One reason for this discrepancy in achievement is that teachers may assume that students transfer skills used in reading story selections to reading word problems and, therefore, may overlook the role that reading comprehension plays in mathematical problem solving. Unfortunately, many educators emphasize producing the right answer using the correct algorithm, often ignoring developing the reading comprehension and thinking skills necessary for students to become efficient word problem solvers.

Another reason for this discrepancy in achievement might be attributed to the current organizational structure of many classrooms. Each major content area in the elementary curriculum has become compartmentalized. Whether these structures were created for ease of instruction, mandated time allotments, administrative edict, or organizational patterns, the result is the same. Language arts, reading, mathematics, science, and social studies are taught in discrete time blocks, seldom--if ever--exploring the relationship of thinking processes and pedagogical principles that exist among the language arts and the content areas.

A structured strategy advocated by many reading specialists in the Directed Reading Activity (DRA). Research reported by Bean and Pardi (1979) and others (Cunningham, Moore, Cunningham, & Moore, 1983) indicated that the DRA is an effective teaching strategy. The six major components of this strategy are: (a) developing vocabulary, (b) building background and motivation, (c) setting the purpose for reading, (d) guiding silent reading, (e) questioning to determine comprehension, and (f) rereading the selection orally. The teacher-directed strategy outlined in the DRA can be applied to the teaching of mathematical word problem solving, resulting in a directed mathematics activity that leads students through the successful solving of word problems (presupposing that students possess the necessary computational skills). This process should serve to establish a solid base for independent problem-solving skills.

Developing Vocabulary

Syntax and vocabulary have been found to be determiners of difficulty in verbal arithmetic problems (Moyer, Moyer, Sowder, & Threadgill-Sowder, 1984). Like story selections, mathematical word problems have new words that need to be taught. This is an especially essential component of understanding word problems as mathematics has vocabulary that is both unique and specific. Some words that might be used interchangeably in common language, such as equal and equivalent or number and numeral, do not have the same mathematical meanings. For students to be able to solve word problems effectively, they must understand the mathematical definitions of all the words in the problem, differentiating between common language usage of a word and its mathematical meaning.

A problem which demonstrates the confusion that can result when vocabulary is ambiguously interpreted is, "How many seconds are there in a year?" The standard answer, "31,536,000 seconds," is derived by multiplying 60 seconds by 60 minutes by 24 hours by 365 days. Students who are divergent thinkers may calculate the answer as 12--January 2nd, February 2nd, March 2nd. . . . The correct answer depends on how the problem solver interprets the meaning of the word seconds.

In the upper grades, problems sometimes contain only words with no numerals, making them even more complex for students. For example, "Two times a number plus eight equals forty." Comprehension of this problem requires understanding the words times, plus, and equals, as well as the correct order of operations. In order to solve this problem, students need to translate the written words to their numerical and symbolic counterparts. In this case, it is $(2 \times N) + 8 = 40$. Without these skills, students are unable to translate the written language to a mathematical sentence to solve the problem.

A typical problem for fourth graders is, "Mr. Henry wanted to build a fence around his rectangular garden which was 11 feet by 16 feet. How many feet of fence does he need?" In this problem, the mathematical meanings of feet and rectangular must be understood to find the correct answer. The teacher can elicit the meanings from the students or supply them using visual examples (a 12-inch ruler to demonstrate one foot and a diagram to demonstrate rectangular). After the words have been defined, students should be referred back to the words in the context of the problem.

Building Background and Motivation

After it has been ensured that students understand all the vocabulary in the problem, it is important to build background and motivation. Ask students to describe similar problems they have encountered in their lives. Also, an effective teacher practice is to reword the problem using the students' names, substituting locations familiar to the students, and changing the objects to items that currently interest the students, thereby building upon the background the students bring to the classroom. For example, the problem could be rephrased to read, "Mark and his father wanted to build a fence around their rectangular back yard to keep in their dog. The yard was 11 feet by 16 feet. How many feet of fence do they need?"

Another technique which can serve as a motivating strategy as well as assisting students in solving word problems is to have the students draw a picture of the problem. In the prior problem, some students would either add or multiply the two numbers, producing an incorrect response in both cases; however, by illustrating the problem, students can visually reinforce the concept of perimeter which would allow them to apply the formula of $P = (2 \times W) + (2 \times L)$ and process the numbers correctly. Visual representation lowers reading-related memory overload, enhances semantic representation, combines images and memory structures, and aids problem information organization (Moyer, Sowder, Threadgill-Sowder, & Moyer, 1984). In addition, having students draw pictures of problems can provide the teacher with a diagnostic tool for evaluating their understanding (or misunderstanding) of mathematical concepts and processes.

Setting the Purpose

During this step, the teacher directs the students' attention to the task at hand. Setting the purpose for reading the problem can be accomplished by telling the students to, "Read to find the question that this problem asks," or "Read to find out how many feet each side of the yard is." In setting the purpose, the teacher helps to focus the children's attention by directing them to look for specific information in the problem.

Guiding Silent Reading/Guided Practice

In order to involve all the students in the process, the teacher should direct them to silently read the problem and work the solution on their papers. (This procedure is preferable to having one student work the problem on the board with the other students passively observing.) After all the students have had the opportunity to compute an answer, the teacher should ask one of the students to give the answer with an explanation of how he or she arrived at the solution. This guided practice with immediate feedback should serve to reinforce appropriate problem-solving skills.

Questioning to Determine Comprehension

In solving word problems, students often, out of frustration, look for numerals and apply some random mathematical operation to arrive at an answer, not necessarily the right one, because they lack comprehension of the question. In this phase, ask students to supply more than one formula to derive the correct answer. For example, $P = (2 \times W) + (2 \times L)$; $P = W + W + L + L$; $P = (2 \times L) + (2 \times W)$. This helps students to understand that there is more than one way to derive the correct answer and promotes divergent thinking.

Also, having students restate problems using their own words helps them produce a clearer mental image of the problem, and it can be a useful diagnostic tool for the teacher when students are having difficulty solving problems.

Other comprehension questions can be generated to help the students analyze the written passages and correctly interpret the specialized vocabulary of mathematics. One way of doing this is to rephrase problems using a different formula. For example, "Would Mark and his father need the same amount of fence if their yard was 13 1/2 feet by 13 1/2 feet?"

In some elementary textbooks, pages of word problems deal only with the specific computational skill currently being taught. Students can figure this out rather easily, and consequently, determining the operation to be used to solve the problems is no longer required. They can simply process the numbers of each problem based on the way they solved the beginning problem in the set. In addition to changing the wording, the teacher might also change the computational skills required to solve the problem, for example. "If Mark and his father had 200 feet of fence, how much would be left over after building a rectangular fence around a yard that was 11 feet in width and 16 feet in length?"

Rereading

The final step is rereading the problem. The primary function of this step is for the students to determine if the solution computed makes sense. Students need to address questions such as, "Did I answer the question that was asked? Was my estimate reasonable? Did my drawing portray the problem?" In other words, students need to verify whether their answer is logical.

During this verification step, teachers are encouraged to have students use calculators as tools for performing rapid calculations. Though many educators have expressed the fear that use of calculators will diminish students' abilities to process algorithms with paper and pencil, comprehensive reviews of research in this area reported by Suydam (1979) and Roberts (1980) refute this. The majority of the studies reported in these reviews indicated that students who used calculators did equally well or better on the traditional paper-and-pencil algorithm tests than did children who had not used calculators in their daily work. Teachers may wish to allow students to use calculators during the guided practice as well, perhaps alternating the use of calculators with calculations by hand, once the algorithms are mastered.

Summary

Problem solving is inherently interesting to most children. Because of this, it can be used to enhance students' inductive and deductive reasoning powers and develop comprehension skills as well as helping to develop critical thinking. With a minimum of additional planning and creativity, teachers can implement an added dimension to their instruction of this skill. When applying the strategy of the DRA to mathematical problem solving, the processes involved--rather than merely performing computations--become the focus of the lesson. The strength of this strategy is that it is teacher directed, leading students in interactive comprehension of the process and outcome that build independent problem-solving skills.

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